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# HEAT AND MASS TRANSFER IN CONDENSATION OF WATER VAPOR FROM MOIST AIR

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*Results of an analysis of heat and mass transfer in condensation of vapor from moist air are presented. The computational model is based on the solution of integral relations of boundary-layer energy and diffusion using the analogy of heat- and mass-transfer processes. The effect of the temperature and concentration boundary conditions on the relation of the components of the heat flux on the wall is analyzed. Results of calculations are compared to available experimental data.*

**Introduction.** In air-conditioning systems, in cooling of power plants, and in refrigeration engineering, one often encounters processes of condensation of water vapor from moist air. Here, the heat- and mass-transfer processes are interrelated, and the total heat flux from the gas-vapor mixture is composed of convective heat transfer and the heat of phase transition. As previous investigations of the heat transfer of moist air in channels showed [1-5], these components of the total heat flux are comparable; therefore we need to solve the heat problem simultaneously with the diffusion problem.

The conclusions of available experimental works on the heat and mass transfer of moist air in channels are conflicting. Thus, according to the data of [1-3], heat- and mass-transfer processes are similar and are described by the regularities for flows without phase transitions. Conversely, in [4, 5], the analogy of heat and mass transfer is not confirmed and heat transfer with flow of moist air is substantially intensified as compared to flow along a "dry" wall. Possible causes of this difference are the effect of cross flow of the vapor at the wall and the increase in the interphase heat exchange surface due to condensate drops forming on the wall. Therefore development of this field requires new experimental investigations.

The present work is devoted to a computational analysis of combined heat and mass transfer in flow of moist air accompanied by condensation of vapor on a wall. The problem is solved in a simplified formulation using integral transport equations and the similarity of heat- and mass-transfer processes. This makes it possible to obtain simple calculational relations and to analyze in detail the effect of different factors on the heat and mass transfer, having revealed the governing factors among them. This approach is quite efficient and yields, as will be shown below, qualitative agreement with experiment.

**Formulation of the Problem. Basic Equations.** The flow diagram is shown in Fig. 1. A plane surface is washed in the longitudinal direction by a binary air-vapor mixture with the pressure $p_0$, the temperature $T_0$, and the weight concentration of the vapor $K_{10}$. For simplicity we assume that the temperature and the composition of the mixture are constant along the length. By virtue of the conservatism of the laws of heat and mass transfer toward a change in the boundary conditions relations obtained under these conditions can be used subsequently for analysis of more complicated cases, such as, for example, flow in the initial and stabilized segments of a tube.

We disregard the effect of the condensate forming on the surface, considering its thermal resistance to be negligibly small. This assumption is justified because in the majority of the experiments conducted on the heat transfer of moist air in channels [2-6] a dropwise regime of condensation was observed. Drops forming on the cooled surface are periodically removed by the flowing air stream. Then the surface temperature takes the saturation value $T_w = T^*$ for the vapor concentration on the surface $K_{1w}$.

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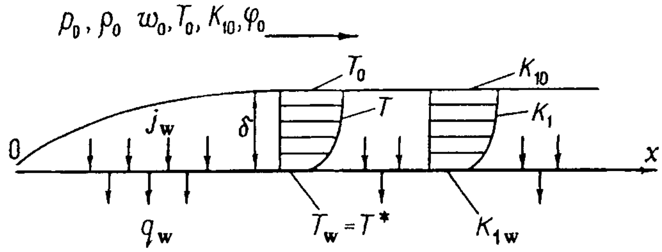



Fig. 1. Diagram of flow in condensation of an air-vapor mixture.

The sought parameters in this problem are the distributions of the heat- and mass-transfer coefficients and accordingly the heat and mass fluxes along the surface in the flow. The processes of heat and mass transfer in moist air are interrelated, and therefore we need to solve the energy and diffusion equations simultaneously, using in addition the momentum conservation equations.

The energy equation for the binary gas mixture in the boundary layer, written in enthalpies, has the form

$$\rho w_x \frac{\partial i}{\partial x} + \rho w_y \frac{\partial i}{\partial y} = \frac{\partial}{\partial y}\left\{\frac{\lambda}{\bar{c}_p}\left[\frac{\partial i}{\partial y} + (\mathrm{Le} - 1)(i_1 - i_2)\frac{\partial K_1}{\partial y}\right]\right\}, \tag{1}$$

and the diffusion equation for the vapor is

$$\rho w_x \frac{\partial K_1}{\partial x} + \rho w_y \frac{\partial K_1}{\partial y} = \frac{\partial}{\partial y}\left(\rho D \frac{\partial K_1}{\partial y}\right). \tag{2}$$

Since there are no phase transitions within the boundary layer, the enthalpy of the gas mixture in Eq. (1) is

$$i = \sum_i K_i \int_0^T c_{pi}\, dT .$$

For the binary system and in the temperature range considered ($t < 100^\circ$C) $c_{pi} \approx$ const and the expression for the enthalpy can be represented as

$$i = \bar{c}_p T = [c_{p1} K_1 + c_{p2} K_2]\, T , \tag{3}$$

where $\bar{c}_p$ is the average heat capacity of the air-vapor mixture at the point of the boundary layer considered.

The Lewis number $\mathrm{Le} = \rho \bar{c}_p D/\lambda$ for the mixture of water vapor with air when the vapor concentrations are low ($K_1 \to 0$) is close to unity ($\mathrm{Le} \approx 1.1$), and therefore we can disregard the second term in the right-hand side of Eq. (1). Then the energy equation becomes similar to the diffusion equation. Consequently, the integral relations of energy and diffusion will be similar, too, and for gradient-free flow they will coincide with the momentum conservation equation.

Upon integrating (1) simultaneously with the continuity equation we have

$$\frac{d\mathrm{Re}_i^{**}}{d\bar{x}} + \frac{\mathrm{Re}_i^{**}}{\Delta i}\frac{d\Delta i}{d\bar{x}} = \mathrm{St}\,(1 - b_{1t})\,\mathrm{Re}_L . \tag{4}$$

Here $\mathrm{Re}_i^{**}$ and $\mathrm{Re}_L$ are the Reynolds numbers, calculated from the energy loss thickness and the characteristic linear scale; $b_{1t}$ is the thermal (enthalpy) permeability parameter; $\mathrm{St} = \Psi \mathrm{St}_0$ is the Stanton number; $\Psi$ is the heat- and mass-transfer function, which characterizes the action of disturbing factors on the laws of transfer (the cross vapor flow (suction) on the surface, nonisothermicity, inhomogeneity of the composition, etc.); $\Delta i = i_0 - i_w$ is the difference in the enthalpies between the flow core and the wall.

The integral diffusion equation has a form similar to (4) when $\mathrm{Re}_i^{**}$, $b_{1\mathrm{t}}$, $\Delta i$, and St are replaced by the corresponding diffusion analogs. Then, allowing for the similarity of the processes of heat and mass transfer and friction, we reduce the solution of the problem to determination of the thermal and diffusion parameters of the permeability and calculation, from Eq. (4), of the corresponding diffusion relationship of all the quantities sought.

The permeability parameters are determined from the equation of conservation of matter and energy on the wall. For the vapor flow on the surface, allowing for the fact that the wall is impermeable to air, we have

$$j_\mathrm{w} = j_\mathrm{w}(K_1)_\mathrm{w} - \rho D\left(\frac{\partial K_1}{\partial y}\right)_\mathrm{w}. \tag{5}$$

With allowance for the fact that the diffusion Stanton number is equal to

$$\mathrm{St_d} = -\rho D\left(\frac{\partial K_1}{\partial y}\right)_\mathrm{w} \Big/ \Delta K_1 \rho_0 w_0, \tag{6}$$

we obtain an expression for the diffusion parameter of the permeability:

$$b_{1\mathrm{d}} = \frac{(K_1)_0 - (K_1)_\mathrm{w}}{1 - (K_1)_\mathrm{w}}. \tag{7}$$

The heat balance on the surface can be written as

$$q_\mathrm{w} = \left(-\lambda\frac{\partial T}{\partial y}\right)_\mathrm{w} + j_\mathrm{w} r. \tag{8}$$

Here $q_\mathrm{w}$ is the total heat flux removed through the wall; $(-\lambda\partial T/\partial y)_\mathrm{w} = q_\mathrm{c}$ is the heat flux supplied by convection from the gas-vapor mixture to the wall, and $j_\mathrm{w}r = q_j$ is the heat consumed by vapor condensation. Then we represent Eq. (8) as the sum of the components of the heat flux on the wall:

$$q_\mathrm{w} = q_\mathrm{c} + q_j. \tag{9}$$

The thermal parameter of the permeability is written as $b_{1\mathrm{t}} = j_\mathrm{w}/\rho_0 w_0 \mathrm{St_c}$, and the Stanton number in it is determined from the convective component of the heat flux: $\mathrm{St_c} = q_\mathrm{c}/\rho_0 w_0 \Delta i$. With allowance for this fact, from Eq. (9) we obtain an expression for the thermal parameter of the permeability:

$$b_{1\mathrm{t}} = \frac{\Delta i}{r}\left(\frac{q_\mathrm{w}}{q_\mathrm{c}} - 1\right) = \frac{1}{\mathrm{K}}\left(\frac{\mathrm{St}_\Sigma}{\mathrm{St_c}} - 1\right), \tag{10}$$

where K is the Kutateladze number and $\mathrm{St}_\Sigma = q_\mathrm{w}/\rho_0 w_0 \Delta i$ is the Stanton number, calculated from the total heat flux on the wall.

By virtue of the adopted analogy of heat and mass transfer we can write [7]

$$b_{1\mathrm{t}} = b_{1\mathrm{d}}\,\mathrm{Le}^{0.6}. \tag{11}$$

By simultaneously solving Eqs. (7), (10), and (11) we obtain an expression for the ratio of the components of the heat flux on the wall:

$$\frac{q_\mathrm{w}}{q_\mathrm{c}} = \left(1 + \frac{q_j}{q_\mathrm{c}}\right) = 1 + \mathrm{Le}^n b_{1\mathrm{d}}\mathrm{K} = 1 + \mathrm{Le}^n\frac{(K_1)_0 - (K_1)_\mathrm{w}}{1 - (K_1)_\mathrm{w}}\mathrm{K}. \tag{12}$$

Thus, Eq. (12) together with the dependence for the saturation curve enables us, for a prescribed ratio of the heat-flux components, to calculate the temperature, concentrations, and enthalpy of the gas-vapor mixture on

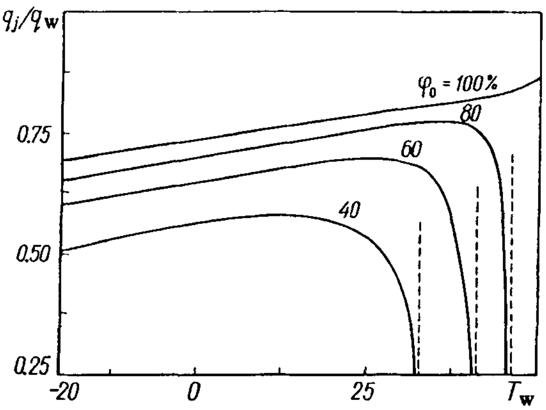



Fig. 2. Ratio of components of heat fluxes on a surface in moist-air flow. $T_w$, °C.

the wall and the permeability parameters $b_{1t}$ and $b_{1d}$. Similarly we solve the inverse problem when the wall temperature is prescribed and the ratio of the heat fluxes is calculated. It is important that the relative components of the heat fluxes on the wall $q_w/q_c$ and $q_j/q_c$, as expression (12) shows, do not depend on the aerodynamics of the problems in question. The flow regime has a very slight effect on the results of the calculation since, with laminar flow, the exponent on the Lewis number is $n = 0.66$ and with turbulence $n = 0.6$. This fact enables us to analyze quite simply and efficiently the effect of the boundary conditions on the ratios of the components of the heat fluxes on surfaces with phase transitions.

For calculations of the distributions of the heat- and mass-transfer coefficients and the heat and mass fluxes along the length of the surface in the flow, we need to solve the integral energy (4) and diffusion relations. For the turbulent regime of flow along the surface, the solution of the integral energy relation has the form [7]

$$\mathrm{St_d} = 0.029\,\mathrm{Re}_x^{-0.2}\,\Psi_t^{0.8}\,(1 - b_{1d})^{-0.2}\,\mathrm{Sc}^{-0.6}\,, \tag{13}$$

while the convective heat-flux component is determined as $q_c = \mathrm{St}_d\rho_0 w_0 \Delta i$. The total heat flux is then calculated by formula (12).

The mass velocity of the vapor condensed on the wall is found in a similar manner from the solution of the diffusion equation. In the final analysis, we have the calculational dependence

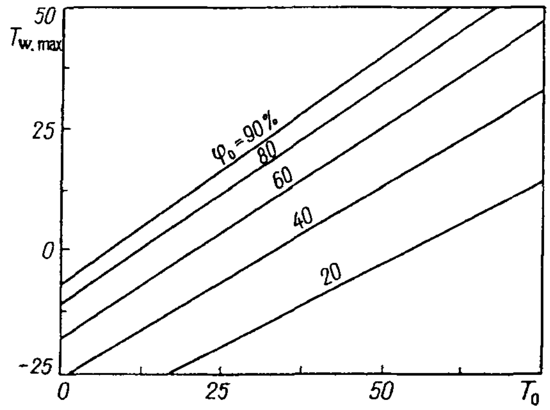
$$j_w = b_{1d}\,\mathrm{St_d}\,\rho_0 w_0 = 0.029\,\mathrm{Re}_x^{-0.2}\,b_{1d}\rho_0 w_0 \Psi_d^{0.8}\,(1 - b_{1d})^{-0.2}\,\mathrm{Sc}^{-0.6}\,, \tag{14}$$

which enables us to find the distribution of the cross vapor flow along the length of the surface in the flow.

The functions of heat and mass transfer $\Psi_t$ and $\Psi_d$, which characterize, in (13) and (14), the action of the processes of vapor suction and the nonisothermicity and the variability of the composition across the thickness of the turbulent boundary layer on the heat- and mass-transfer intensity, can be determined by the formulas of [8] for selective suction. However an analysis showed that, in the region of moderate temperatures of the moist air ($t_0 \leq 50$°C), the intensity of the suction is low, and its effect does not exceed 10–15%. Therefore, as a first approximation, we assume that $\Psi_t \approx \Psi_d = 1$. In this case, relations (13) and (14) are most simplified and take a form that describes heat and mass transfer for quasi-isothermal gas flow along an impermeable plate.

**Discussion of the Results of Calculation. Comparison with Experiment.** Calculations were performed in a wide range of the parameters in the flow core ($T_0$ = 10–75°C, $\varphi_0$ = 0–100%, $p_0 = 1\cdot10^4$–$10^6$ Pa) and on the wall ($T_w$ = −20–50°C). Thermophysical properties were determined from the parameters of the wall, the mixture enthalpy and the heat of phase transition were taken from tabulated data [9], and the Lewis number as a function of the vapor concentration was taken from [10].

Results of a calculation of the components of the heat flux $q_j/q_w$ as functions of the wall temperature for different relative humidities are shown in Fig. 2. The dashed lines here correspond to the dew-point temperature, at which condensation on the wall becomes evaporation. It can be seen that the calculated curves are substantially



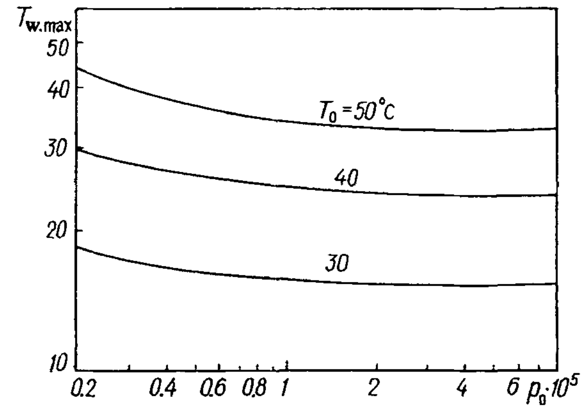




Fig. 3. Thermal regimes with the maximum $q_j/q_w$ for heat and mass transfer of moist air; $p_0 = 10^5$ Pa.

Fig. 4. Effect of the pressure of a gas-vapor mixture on the regime of heat and mass transfer; $\varphi_0 = 80\%$.

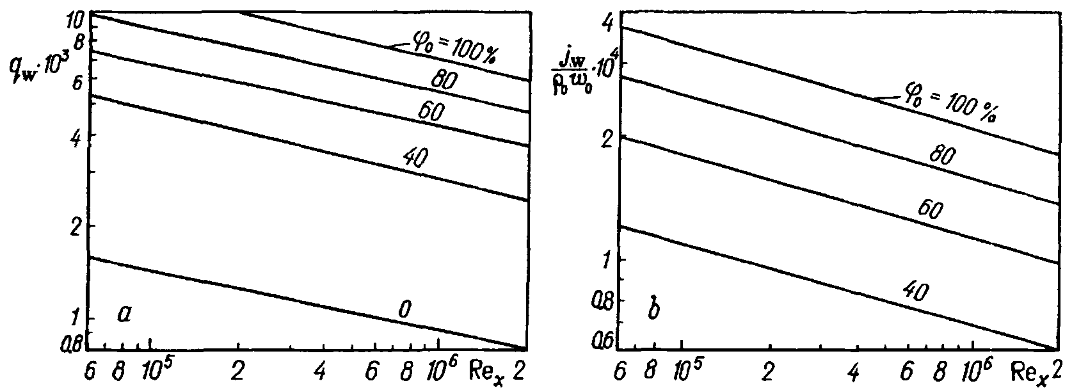



Fig. 5. Heat (a) and mass (b) fluxes of vapor vs. the Reynolds number in turbulent moist-air flow; $p_0 = 10^5$ Pa, $T_0 = 50°C$, and $T_w = 10°C$.

stratified as the moisture content of the flow changes, and as it increases, the contribution of the heat of phase transition to the total heat flux on the wall becomes prevailing. The change in $q_j/q_w$ as a function of the wall temperature is nonmonotonic in character and, for some values of $T_w$, it attains a maximum, and, in this case, the largest mass flux of condensed vapor per unit heat energy removed from the wall will occur. The heat flux transferred by convection will be smallest here. The extremum character of the transition in relative heat fluxes on surfaces with phase transitions was shown in [11]; however, for the case of condensation, it was not analyzed in detail.

The regime of the maximum ratio $q_j/q_w$ depends on the partial pressure of the mixture's vapor and the temperatures in the flow and on the wall and can be calculated from relation (12) when it is investigated for a maximum. Results of this investigation are presented in Fig. 3 for atmospheric pressure and in Fig. 4 for different pressures of the gas-vapor mixture. We note that the relative humidity has a substantial effect on the temperatures at which maximum vapor condensation is attained. At the same time, the pressure of the gas-vapor mixture, as follows from Fig. 4, has a very slight effect on the wall temperatures at which $q_j/q_w$ is maximum.

It is important to emphasize that the data of Figs. 2-4 can be used for both laminar and turbulent regimes of flow. They make it possible to analyze quite simply the relationships between the components of the heat flux on the wall and to reveal basic features of heat- and mass-transfer processes for moist air. Calculation of data for other parameters of the air-vapor mixture presents no fundamental problems and can be performed by the above procedure.

Results of a calculation of the heat and mass fluxes on the wall for turbulent moist-air flow along an isothermal surface and for a variation in the relative humidity are presented in Fig. 5a and b, respectively. As can

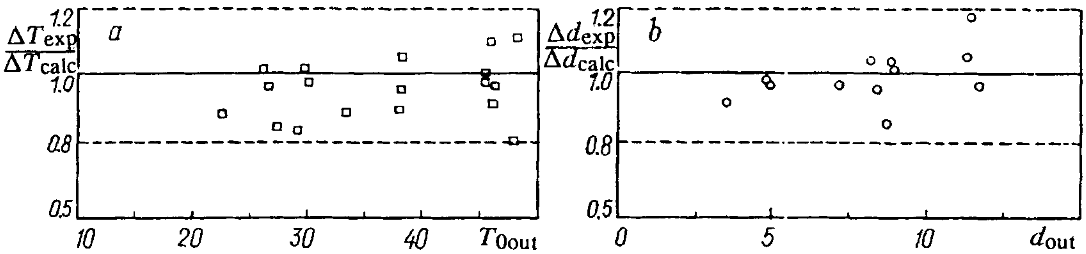



Fig. 6. Correlation between calculated data and V. N. Patrikeev's experiments on the difference of the temperature (a) and the moisture content of vapor (b) between the inlet and outlet of the channel.

be seen, an increase in $\varphi_0$ leads to a substantial increase in the heat and mass fluxes. Thus, the intensification of the heat transfer in saturated air attains practically one order of magnitude relative to dry-air flow. A particularly strong effect of the moisture content is observed in the region $\varphi_0 \to 0$.

It proved difficult to compare data of the calculation to available experiments because of a lack of complete experimental information in the works. For this reason, in the comparison, use was made of experimental data presented by V. N. Patrikeev on heat and mass transfer of moist air in a cylindrical tube at elevated pressures. The diameter of the channel was 8 mm, the length was 125 diameters, and the Reynolds number $\mathrm{Re}_{D_{ch}} \approx (1-2) \cdot 10^5$. A wide range of pressures $p_0 = 0.2-0.7$ MPa of the air-vapor mixture and temperatures of the flow $T_0 = 30-60^{\circ}$C and the wall $T_w = 10-40^{\circ}$C was covered by the experiments. The calculations were performed using prescribed conditions at the inlet to the channel and measured lengthwise distributions of the wall temperature. Since, with flow in the channel, the moisture content and temperature of the mixture change along the length, the entire channel was divided into 200 segments, and the calculation was performed step-by-step from the inlet cross section to the outlet cross section.

The condensation regime was considered to be dropwise, as demonstrated by visual observations, and therefore the vapor concentration corresponded to saturation for the prescribed wall temperature. Allowing for the difficulty of obtaining local values of the heat and mass fluxes in the experiment and the considerable errors that occur in averaging these quantities when the heat flux, the moisture content, and the temperature in the core and on the wall change along the length, all calculations and experiments are compared for integral parameters at the inlet and outlet of the channel. These quantities are directly measured in the experiments and are the most reliable.

Results of the comparison in the form of the ratio $\Delta T_{exp}/\Delta T_{calc}$ as a function of the temperature and $\Delta d_{exp}/\Delta d_{calc}$ as a function of the moisture content at the outlet are shown in Fig. 6. Here $\Delta T = T_{in} - T_{out}$ and $\Delta d = d_{in} - d_{out}$ are the average temperature and concentration differences between the inlet and outlet of the channel over the cross section. As can be seen, the disagreement between the calculated and experimental data lies in a region bounded by an error that does not exceed $\pm 20\%$. This demonstrates the legitimacy of the basic and simplifying assumptions that form the basis of the computational procedure.

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## NOTATION

$b_{1d}$ and $b_{1i}$, permeability parameters; $c_p$, specific heat at constant pressure, J/(kg·deg); $\bar{c}_p = c_{p1}K_1 + c_{p2}(1 - K_1)$; $D$, diffusion coefficient, m$^2$/sec; $D_{ch}$, channel diameter, m; $d$, moisture content of the air, g/kg; $i$, enthalpy of the gas mixture, J/kg; $j$, mass flux, kg/(m$^2$·sec); $K_{1,2}$, weight concentrations of the components, kg/kg; K, Kutateladze number; $r$, heat of phase transition, J/kg; $\mathrm{Re}_x = w_0 x/\nu$, $\mathrm{Re}_L = w_0 L/\nu$, $\mathrm{Re}_i^{**} = w_0 \delta_i^{**}/\nu$, Reynolds numbers; $q$, heat flux, W/m$^2$; $\mathrm{Sc} = \nu/D$, Schmidt number; $\mathrm{St} = q_c/\rho_0 w_0 \Delta i$, Stanton number; $T$, temperature, deg; $w$, velocity, m/sec; $\nu$, kinematic viscosity, m$^2$/sec; $\varphi_0$, relative humidity in the flow core; $\Psi$ =

$St/St_0$, relative function of heat and mass transfer. Subscripts and superscripts: 0, flow core, standard conditions; 1, vapor; 2, air; w, wall; $^{**}$, saturation conditions; d, diffusion; t, enthalpy (thermal).

## REFERENCES


1. Yu. N. Pchelkin, Teploénergetika, No. 6, 72-75 (1961).
2. V. M. Semein, Teploénergetika, No. 4, 11-15 (1956).
3. A. M. Baklastov and Zh. F. Sergazin, Izv. Vyssh. Uchebn. Zaved., Énergetika, No. 2, 59-64 (1965).
4. B. M. Smol'skii, P. A. Novikov, and L. A. Shcherbakov, Inzh.-Fiz. Zh., **21**, No. 1, 71-74 (1971).
5. P. A. Novikov and L. A. Scherbakov, Inzh.-Fiz. Zh., **23**, No. 4, 737-742 (1977).
6. A. N. Khoze and V. N. Patrikeev, Thermal Physics and Hydrogasdynamics of Boiling and Condensation Processes, Vol. 3. Condensation, Part 1 [in Russian], Riga (1986), pp. 154-161.
7. S. S. Kutateladze and A. I. Leont'ev, Heat and Mass Transfer and Friction in a Turbulent Boundary Layer [in Russian], Moscow (1985).
8. É. P. Volchkov and V. I. Terekhov, Inzh.-Fiz. Zh, **44**, No. 2, 115-121 (1983).
9. M. P. Vukalovich, Thermophysical Properties of Water and Water Vapor [in Russian], Moscow (1967).
10. V. M. Keis, Convective Heat and Mass Transfer [in Russian], Moscow (1972).
11. B. F. Boyarshinov and V. I. Terekhov, Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekhn. Nauk, Issue 1, 25-31 (1986).